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Energy Flux of a Continuous Acoustic Plane Wave Train

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For a plane acoustic wave, potential and kinetic energy are equal (Lighthill (1978), p. 13). We can utilize this fact to leverage the expression for acoustic kinetic energy averaged across an acoustic impulse

$$I = \frac{\rho_0 c}{\tau} \int_0^\tau u(t)^2 dt \tag{1}$$

where I is acoustic intensity (Watts per square meter), ρ_0 is ambient air density, c is the speed of sound, τ is the signal duration, and $u(t)$ is particle velocity (see Lighthill (1978), page 13, Krasnov et al. (2007), equation 2). We can calculate the particle velocity in an acoustic plane wave via Equation 17 in Lighthill (1978):

$$p - p_0 = \rho_0 c u(t) \tag{2}$$

where p is the over/underpressure and p_0 is ambient pressure. For a sinusoidal disturbance, then, the particle velocity fluctuates as

$$u(t) = u \sin \omega t \tag{3}$$

Given all this, we can calculate the average acoustic intensity across one cycle of the wave:

$$I = \frac{1}{2} \rho_0 c u^2 \tag{4}$$

Substituting for u in Equation 2 and some manipulation gives

$$I = \frac{1}{2} \frac{(p - p_0)^2}{c \rho_0} \tag{5}$$

or, in terms of root mean square amplitudes (Jensen et al., 2011)

$$I = \frac{p_{rms}^2}{\rho_0 c} \tag{6}$$

Now we have calculated the energy flux per *cycle* of our sinusoidal source: this is the physics definition of "intensity". Multiplying by the square of frequency ω of the wave will give energy flux per unit time:

$$\tilde{E} = \frac{1}{2} \frac{\omega^2}{c \rho_0} (p - p_0)^2 \tag{7}$$

which is what I am after. A similar form can be seen for seismic energy flux as well (Shearer (2009), Equation 6.15)

$$\tilde{E} = \frac{1}{2} c \rho_0 u^2 \omega^2 \tag{8}$$

Thus, Equation 7 is physically consistent and alluded to in a couple of sources. Energy scales with frequency, so the higher the frequency, the higher the energy. Obviously a higher pressure amplitude gives a higher energy. Furthermore, amplitudes of waves in two different media are proportional to the ratios of the square roots of their impedances: $\sqrt{\rho_0 c}$. Thus, a lower density for a given pressure actually implies a more energetic wave. This follows from kinetic considerations of course.

At first, glance, Equation 1 of Barry et al. (1966) appears to give a different result than using Equation 7 in this text. Their equation

$$P = \frac{\rho_0 c^3}{2\gamma^2}\left(\frac{p_{max} - p_0}{p_0}\right) \tag{9}$$

considers power density P, adiabatic index γ, maximum overpressure p_{max} and ambient pressure p_0 *per unit wavelength*. This is related to Rayleigh (1894), Section 245, Equation 10:

$$W = \frac{1}{2}\rho_0 c^3 u_{max}^2 t \tag{10}$$

Taking the time derivative of the above and dividing by the square of wavelength $\frac{\omega}{c}$ yields

$$\tilde{E} = \frac{1}{2}\omega^2 \rho_0 c u^2 \tag{11}$$

identical to seismic energy flux (Equation 8).

Thus, the formula for the amplitude change of a pressure wave with overpressure p_0 at ρ_0 and c_0 and p_1 at ρ_1 and c_1 is

$$p_1 = p_0\sqrt{\frac{c_1\rho_1}{c_0\rho_0}} \tag{12}$$

in other words, the square root of the reciprocal of their acoustic impedances. The practical consequence for this is the following: a nondissipative, harmonic acoustic plane wave will have approximately 10x lower pressure amplitude in the middle stratosphere compared to the same wave measured at sea level. This is primarily due to the approximately 2 orders of magnitude lower air density in the stratosphere compared with the surface.

References

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